Modelling macroscopic and baby universes by fundamental strings

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Abstract

We develop a model of $(1+1)$-dimensional parent and baby universes as macroscopic and microscopic fundamental closed strings. We argue, on the basis of understanding of strings from the point of view of target D-dimensional space-time, that processes involving baby universes/wormholes not only induce c-number "α-parameters" in $(1+1)d$ action, but also lead to loss of quantum coherence for a $(1+1)d$ observer in the parent universe.

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1 Introduction

It has been argued some time ago that quantum gravity may allow for processes in which small (say, Planck size) baby universes branch off the large one, and that these processes may, among other things, lead to the loss of quantum coherence in the parent universe [1, 2, 3]. Indeed, a state like $|\psi\rangle \times |0\rangle_{baby}$ would evolve into a state

$$|\psi'\rangle \times |0\rangle_{baby} + |\psi''\rangle \times |1, I\rangle_{baby} \tag{1}$$

where $|\psi\rangle \dots |\psi''\rangle$ refer to the states of the parent universe, and $|\rangle_{baby}$ is the state vector in the Hilbert space of baby universes (i.e., $|0\rangle_{baby}$ is the state with no baby universes, and the state $|1, I\rangle_{baby}$ is the state with one baby universe of the type I). Since baby universes cannot be probed by an observer in the large universe, this observer would interprete the state (1) as one described by a non-trivial density matrix; this would mean an apparent loss of quantum coherence in the large universe.

Coleman [4] and Giddings and Strominger [5] put forward the following argument against this observation. The effects of baby universes on low energy physics in the parent universe may be summarized by adding extra local terms to the lagrangian

$$\Delta L(x) = \sum_I \hat{A}_I O_I(x) \tag{2}$$

where O_I are local operators composed of fields living in the large universe, and $\hat{A}_I$ are x-independent operators acting on states of the baby universe subsystem. It has been argued, furthermore, that the operators $\hat{A}_I$ commute with each other; in an appropriate basis $\hat{A}_I = A_I + A_I^\dagger$ where $A_I^\dagger$ creates a baby universe of the type I from $|0\rangle_{baby}$, and A_I is the corresponding annihilation operator. If so, one can diagonalize the set of operators $\hat{A}_I$ by introducing α-states,

$$\hat{A}_I|\alpha\rangle = \alpha_I|\alpha\rangle$$

where α_I are c-numbers. These α-states are superselection sectors of the theory, and in a given superselection sector the extra terms in the lagrangian become

$$\Delta L(x) = \sum_I \alpha_I O_I(x)$$

This means that quantum coherence is restored, and the only effect of baby universes on the parent universe is the appearance of new coupling constants α_I.

The same conclusion has been reached by Klebanov, Susskind and Banks [6] on the basis of the functional integral formalism. However, further development of this

approach has lead Banks [7] to the following picture: the loss of coherence may not be entirely absent in the closed universe, but suppressed by $\exp(-M_{Pl}^3 V)$ where V is the volume of the large universe. Even though practically indistinguishable in a universe like ours, the conclusions of Coleman and Banks look different in principle; this may be regarded as a signal that the problem is not completely understood.

A natural model for probing this set of ideas is the theory of (fundamental) closed strings viewed as the theory of (1+1)-dimensional universes [8, 9]. It has been realized by Hawking [9] and Lyons and Hawking [10] that in this theory, the α-parameters cannot be regarded as c-numbers; they should rather be viewed as field in mini-superspace, the target space of strings. In other words, local operators analogous to eq.(2) in the covariant operator formalism of string theory have the form

$$A_s^\dagger(Q)\,(\zeta_s^{\mu\nu\cdots}\partial_\alpha X^\mu \partial_\alpha X^\nu \ldots)\,\mathrm{e}^{-iQX} \;+\; \text{h.c.} \tag{3}$$

(hereafter we consider strings in critical dimension and call low lying string states collectively gravitons). Here $\alpha = 0, 1$; $\mu, \nu = 0\ldots, D-1$, D is the dimension of the target space-time ($D = 26$ for bosonic string). $X^\mu(\sigma, \tau)$ is viewed as field operator in (1+1) dimensions, while $A_s^\dagger(Q)$ creates baby universes (gravitons). Gravitons with different target space momenta Q and different polarizations ζ are just different kinds of baby universes; (Q, s) stand for index I in eq.(2); integration over Q and summation over s is assumed in eq.(3). The observation of Hawking [9] corresponds to the fact that operators (3) do not commute with each other; not only the combination $(A+A^\dagger)$ but also $(A-A^\dagger)$ appears in eq.(3). Coleman's argument against the loss of quantum coherence apparently does not work.

This observation provides sufficient motivation to take a closer look, at the tree level of string interactions, into the graviton emission by fundamental string interpreted as branching off of baby universes in $(1+1)d$ theory. An advantage of this model is that one can invoke intuitive understanding of these processes from the point of view of the target space (mini-superspace).

There exists fairly strong evidence [11, 12, 13] that heavy fundamental strings with only low harmonics excited (leading trajectory or alike) behave in flat target space as classical strings whose length in the c.m. frame is of order of their mass,

$$L \sim M$$

(hereafter α' is set equal to 1/2). They decay slowly by radiating classical soft gravitational waves with wavelengths of order $1/L$. These classical strings are long living

objects: the power radiated into gravitational waves is

$$\frac{dM}{dt} = \text{const} \times \kappa$$

where κ is the gravitational coupling in D dimensions, and constant here is of order 1 (actually, it is closer to 100). Thus, it indeed makes sense to treat these particular string states[1] as (1+1)-dimensional universes, even though the necessary formalism has not yet been elaborated in full detail.

Viewed from (1+1) dimensions, the emission of soft gravitational waves in target space gives rise to extra terms in the $(1+1)d$ lagrangian, whose structure becomes

$$\sqrt{g} g^{\alpha\beta} G_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu \ + \ \ldots$$

where $G_{\mu\nu}$ is the classical target space metrics that includes the gravitational waves emitted by the string. This is precisely the picture of classical α-parameters[2]; it is seen to correspond to the c-number approximation of the operators $A_s^\dagger(Q)$ in the approximate description of the emission of gravitons as the radiation of classical gravitational waves. As expected [4, 5, 6], the actual values of the "α-parameters" $G_{\mu\nu}(X)$ are determined by the history of the large universe (string); from the target space point of view this is the history of the radiation of gravitational waves[3].

However, the radiation of classical gravitational waves in target space is not the whole story. The string (parent universe) may eventually emit a graviton of relatively high target space momentum, which will be lost forever for a $(1+1)d$ observer. This event will be a quantum process, it will not be described by classical α-parameters, and from $(1+1)d$ point of view it will lead to the loss of quantum coherence. The magnitude of this effect will be determined by the amplitude of the graviton emission.

In this paper we develop this model of $(1+1)$-dimensional parent and baby universes – macroscopic and microscopic strings – with the main purpose to discuss the loss of quantum coherence as seen by a $(1+1)d$ observer. The study of microscopic strings in uncompactified D-dimensional space time is technically quite complicated. We find it more convenient to consider D-dimensional flat space-time with one spatial dimension, say, X^1, compactified to a large circle of length $2\pi L$. Then, as suggested

[1] Clearly, these states are not the most general states of highly excited strings, but these are the states that may be suitable for modelling large universes.

[2] This correspondence has been understood by many authors, especially in the context of $2d$ quantum gravity, see, e.g., refs.[14, 15].

[3] But not only by this history. $G_{\mu\nu}(X)$ are determined also by other sources of gravity in target space as well as by boundary conditions in target space (superspace).

by Polchinski [16] and Dai and Polchinski [17], the smooth macroscopic closed string state $|\mathcal{P}\rangle$ is naturally constructed as the lowest state that winds around this compact dimension. For the string at rest, its target space momentum is (we use the conventions of ref.[18])

$$\mathcal{P} = (M_0, \mathbf{0})$$

where

$$M_0^2 = 4L^2 - 8$$

i.e., the mass is indeed of order L at large L.

In the existing discussions of possible effects of wormholes [1, 2, 3, 4, 5, 6, 7], an important role is played by the interactions of particles, living in the parent universe, with baby universes that branch off. To model these interactions, we need particle-like excitations of the fields $X^\mu(\sigma, \tau)$ in $(1+1)d$ universe, in the first place. These are conveniently constructed by making use of the DDF operators [19] that create physical excited string states by acting on the smooth state $|\mathcal{P}\rangle$. These operators are characterized by the mode number n, and n/L may be regarded as the bare $(1+1)d$ momentum (equal to bare $(1+1)d$ energy) of a "particle" (DDF operators automatically create dressed "particles" whose total $(1+1)d$ momentum and energy are zero, as it should be in the closed universe). The interesting regime is $L \to \infty$ with n/L fixed and finite. We outline the construction of the smooth string state and its DDF excitations in section 2.

As discussed above, the loss of quantum coherence occurs when collisions of particles in $(1+1)d$ universe induce the creation of a baby universe, i.e., when the macroscopic excited string emits real microscopic sting states into the target space. In sections 3 and 4 we consider the simplest of these processes, namely, those in which both initial and final states of the macroscopic string contain two "particles". One property of the string theory (in flat target space) as the theory of $(1+1)d$ universes is that there exists global quantum number – target space momentum P^μ – and that baby universes (microscopic strings) carry away this quantum number. As discussed in section 2, the DDF operators carry *light-like* target space momentum ΔP^μ, so the total momentum of the excited string is $\mathcal{P}^\mu + \Delta P^\mu$. If the emitted microscopic string is massless (graviton or dilaton), the final macroscopic string state will typically be a DDF excitation above *moving* smooth string; in other words, zero modes of fields $X^\mu(\sigma, \tau)$ will be excited. Indeed, if the final state is again the DDF excitation above

the smooth string at rest, then the graviton momentum Q^μ is

$$Q^\mu = \Delta P^\mu - \Delta P'^\mu \tag{4}$$

where $\Delta P'^\mu$ is the target space momentum associated with the final DDF operators. Equation (4) may hold only when Q^μ, ΔP^μ and $\Delta P'^\mu$ are collinear, otherwise the smooth part of the final string should carry part of the recoil momentum. In the latter case the emission of a baby universe involves the interaction with the entire parent one, so one expects that the corresponding amplitude is suppressed as $L \to \infty$. We will confirm these expectations by explicit calculations in sect.3. What is more important, there is no extra suppression of the emission probability apart from one just discussed.

If the emitted microscopic string is a tachyon, eq.(4) does not require Q^μ, ΔP^μ and $\Delta P'^\mu$ to be collinear. So, one may expect that the corresponding amplitude is non-vanishing at large L in a wider region of phase space. We calculate this amplitude in sect.4 and see that it is indeed finite in the above kinematics in the limit $L \to \infty$. Unfortunately, the case of two-"particle" excitation of the final macroscopic string, which we consider in this paper, is not generic, and the region of the phase space where the emission amplitude is unsuppressed is of zero measure. We think, however that our observations indicate that the total emission probability, and hence the loss of quantum coherence in $(1+1)d$ universe, is unsuppressed when the conservation of global quantum numbers does not require involvement of the entire parent universe into the process of creation of a baby universe.

Section 5 contains concluding remarks.

2 Macroscopic strings

Let one of the spatial dimensions of the target space, X^1, be compactified to a large circle of length $2\pi L$. We consider bosonic closed string theory in critical dimension, in the sector with one string winding once around this compact dimension. In this sector, the operator $X^\mu(\sigma, \tau)$ is decomposed as follows

$$X^\mu(\sigma,\tau) = X^\mu + P^\mu \tau + 2L^\mu \sigma + \frac{i}{2}\sum_{k\neq 0}\frac{1}{k}\left(\alpha_k^\mu \mathrm{e}^{-2ik\sigma_+} + \tilde{\alpha}_k^\mu \mathrm{e}^{-2ik\sigma_-}\right)$$

where

$$L^\mu = (0, 1, 0, \ldots, 0)$$

and other notations follow the conventions of ref.[18] (in particular, σ belongs to $(0,\pi)$). It is convenient to introduce left- and right-moving components of $X^\mu(\sigma,\tau)$,

$$X_L^\mu(\sigma_+) = \frac{1}{2}X^\mu + \frac{1}{2}P_L^\mu\sigma_+ + \frac{i}{2}\sum_{k\neq 0}\frac{1}{k}\alpha_k^\mu \mathrm{e}^{-2ik\sigma_+}$$

$$X_R^\mu(\sigma_-) = \frac{1}{2}X^\mu + \frac{1}{2}P_R^\mu\sigma_- + \frac{i}{2}\sum_{k\neq 0}\frac{1}{k}\tilde{\alpha}_k^\mu \mathrm{e}^{-2ik\sigma_-}$$

where

$$P_L^\mu = P^\mu + 2L^\mu \qquad P_R^\mu = P^\mu - 2L^\mu$$

The Virasoro operators in the sector with one winding string are[18]

$$L_m = \frac{1}{2} : \sum_k \alpha_k^\mu \alpha_{m-k}^\mu : \qquad \tilde{L}_m = \frac{1}{2} : \sum_k \tilde{\alpha}_k^\mu \tilde{\alpha}_{m-k}^\mu :, \qquad m \neq 0$$

with

$$\alpha_0^\mu = \frac{1}{2}P_L^\mu \qquad \tilde{\alpha}_0^\mu = \frac{1}{2}P_R^\mu$$

and

$$L_0 = \frac{1}{8}P_L^2 + \sum_{k>0}\alpha_{-k}^\mu\alpha_k^\mu \qquad \tilde{L}_0 = \frac{1}{8}P_R^2 + \sum_{k>0}\tilde{\alpha}_{-k}^\mu\tilde{\alpha}_k^\mu$$

The ground state of the string winding around the compact dimension, $|\mathcal{P}\rangle$, is the vacuum of oscillators α_k^μ, $\tilde{\alpha}_k^\mu$. It has $\mathcal{P}^1 = 0$ and

$$M_0^2 = -\mathcal{P}_\mu\mathcal{P}^\mu = 4L^2 - 8$$

Let us construct the DDF operators that create and annihilate excited physical states of the winding string. From the $(1+1)d$ point of view, these states are the states of the large universe with particle-like excitations of the fields $X^\mu(\sigma,\tau)$. The construction begins with choosing a light-like vector

$$e^\mu = (1, \boldsymbol{e}) \qquad \boldsymbol{e}^2 = 1 \tag{5}$$

and a set of transverse spatial vectors $\boldsymbol{\xi}^\alpha$, $\alpha = (1,\ldots,D-2)$,

$$\boldsymbol{\xi}^\alpha\boldsymbol{\xi}^\beta = \delta^{\alpha,\beta} \tag{6}$$

$$\boldsymbol{\xi}^\alpha\boldsymbol{e} = 0 \tag{7}$$

Then the DDF operators are defined as follows,

$$a_n^\alpha = \int_0^\pi \frac{d\sigma_+}{\pi}\exp\left[4in\frac{e_\mu X_L^\mu(\sigma_+)}{e_\mu P_L^\mu}\right]\xi_i^\alpha\partial_+X_L^i(\sigma_+) \tag{8}$$

$$\tilde{a}^{\alpha}_{\tilde{n}} = \int_0^{\pi} \frac{d\sigma_-}{\pi} \exp\left[4i\tilde{n}\frac{e_\mu X^\mu_R(\sigma_-)}{e_\mu P^\mu_R}\right] \xi^\alpha_i \partial_- X^i_R(\sigma_-) \tag{9}$$

Note that the properties of e^μ and $\boldsymbol{\xi}^\alpha$ ensure that $e_\mu X^\mu_{L,R}(\sigma_\pm)$ commute with $(eP_{L,R})$ and with $\xi^\alpha_i \partial_\pm X^i_{L,R}(\sigma_\pm)$. Note also that $(eP_{L,R})$, $e_\mu \alpha^\mu_k$ and $e_\mu \tilde{\alpha}^\mu_k$ commute with the DDF operators.

Making use of these properties, it is straightforward to check that the operators (8), (9) obey the usual oscillator commutational relations,

$$[a^\alpha_n, a^\beta_{n'}] = \delta^{\alpha\beta} n \delta_{n+n'}$$

$$[a^\alpha_n, \tilde{a}^\beta_{\tilde{n}}] = 0$$

Their commutational relations with the Virasoro operators can be found after some algebra,

$$[L_m, a^\alpha_n] = -\frac{n}{(eP_L)} a^\alpha_n (e_\mu \alpha^\mu_m), \qquad m \neq 0 \tag{10}$$

$$[\tilde{L}_m, a^\alpha_n] = \frac{n}{(eP_L)} a^\alpha_n (e_\mu \tilde{\alpha}^\mu_m), \qquad m \neq 0 \tag{11}$$

$$[L_0, a^\alpha_n] = -\frac{n}{2} a^\alpha_n \tag{12}$$

$$[\tilde{L}_0, a^\alpha_n] = \frac{n}{2}\frac{(eP_R)}{(eP_L)} a^\alpha_n \tag{13}$$

The commutational relations for $\tilde{a}^\alpha_n$ are obtained from eqs.(10) – (13) by interchanging $a \leftrightarrow \tilde{a}$, $L_m \leftrightarrow \tilde{L}_m$, $P_L \leftrightarrow P_R$, $n \leftrightarrow \tilde{n}$, $\alpha_k \leftrightarrow \tilde{\alpha}_k$.

Even though the DDF and Virasoro operators do not commute with each other, the operators a^α_n and $\tilde{a}^\alpha_{\tilde{n}}$ can be used for constructing physical states of excited string out of the smooth state $|\mathcal{P}\rangle$. Indeed, consider a state

$$|n, \alpha; \tilde{n}, \beta\rangle = \frac{1}{\sqrt{n\tilde{n}}} a^\alpha_{-n} \tilde{a}^\beta_{-\tilde{n}} |\mathcal{P}\rangle \tag{14}$$

where we have chosen the normalization factor in such a way that the norm of this state coincides with the norm of the smooth string state $|\mathcal{P}\rangle$ (in $(1+1)$-dimensional language this corresponds to "one particle per volume L" normalization). Equations (10) and (11), and similar equations for $\tilde{a}^\beta_{\tilde{n}}$ imply

$$L_m |n, \alpha; \tilde{n}, \beta\rangle = \tilde{L}_m |n, \alpha; \tilde{n}, \beta\rangle = 0, \qquad m > 0$$

The remaining Virasoro constraints,

$$(L_0 - 1)|n, \alpha; \tilde{n}, \beta\rangle = (\tilde{L}_0 - 1)|n, \alpha; \tilde{n}, \beta\rangle = 0$$

are satisfied provided that

$$\frac{n}{(e\mathcal{P}_L)} = \frac{\tilde{n}}{(e\mathcal{P}_R)} \tag{15}$$

The latter condition is the only constraint relating the mode numbers to the light-like vector e^μ. This constraint can be rewritten in the following form,

$$n - \tilde{n} = \frac{2(eL)}{(e\mathcal{P})}(n + \tilde{n}) \tag{16}$$

where $(eL) = e_\mu L^\mu = Le^1$. Equation (15) also implies that

$$\frac{n}{(e\mathcal{P}_L)} = \frac{\tilde{n}}{(e\mathcal{P}_R)} = \frac{n + \tilde{n}}{2(e\mathcal{P})} \tag{17}$$

Provided this constraint is satisfied, the state $|n, \alpha; \tilde{n}, \beta\rangle$ is the physical state. It can be viewed as the dressed oscillator state with mode numbers n and $\tilde{n}$. In $(1+1)$-dimensional language this state can be interpreted as describing the large universe with one left-moving "particle" with bare $(1+1)d$ momentum n/L and and one right-moving "particle" with momentum $(-\tilde{n}/L)$. One can construct physical states with more "particles" in a similar way.

The global quantum number – target space momentum – carried by these excitations can be read out from the commutational relations of the DDF operators with P^μ,

$$[P^\mu, a_n^\alpha] = \frac{2n}{(eP_L)} e^\mu a_n^\alpha$$

$$[P^\mu, \tilde{a}_{\tilde{n}}^\alpha] = \frac{2\tilde{n}}{(eP_R)} e^\mu \tilde{a}_{\tilde{n}}^\alpha$$

These relations mean that the target space momentum carried by the operators a_n^α and $\tilde{a}_{\tilde{n}}^\alpha$ is light-like, $\Delta P^\mu \propto e^\mu$. In particular, the momentum of the state $|n, \alpha; \tilde{n}, \beta\rangle$ is (see eq.(17))

$$P^\mu = \mathcal{P}^\mu - \frac{2(n + \tilde{n})}{(e\mathcal{P})} e^\mu \tag{18}$$

Note that the mass of this excited string state is

$$M^2 = -P^2 = M_0^2 + 4(n + \tilde{n}) \tag{19}$$

which confirms the interpretation of this state in terms of dressed oscillators.

3 Emission of microscopic strings with excitation of zero modes

3.1 Emission of graviton

As discussed in Introduction, $(1+1)d$ baby universes are modelled by low lying string states. Let us first consider the emission of massless states – gravitons (and dilatons). We are interested in the following amplitudes,

$$\kappa\langle f| : \zeta_{\mu\nu}\partial_+ X^\mu(0)\partial_- X^\nu(0)\mathrm{e}^{-iQX(0)} : |i\rangle \tag{20}$$

where initial and final states $|i\rangle$ and $|f\rangle$ are the DDF-excited states of the large string, Q^μ and $\zeta^{\mu\nu}$ are the graviton target space momentum and polarization. In general, the DDF operators corresponding to the initial and final states may be different: they may be constructed with the use of different light-like vectors e^μ and e'^μ and different transverse vectors $\boldsymbol{\xi}^\alpha$ and $\boldsymbol{\xi}'^\alpha$. Thus, in general,

$$|i\rangle = \frac{1}{\sqrt{n\tilde{n}}} a^\alpha_{-n}\tilde{a}^\beta_{-\tilde{n}}|\mathcal{P}\rangle \tag{21}$$

$$|f\rangle = \frac{1}{\sqrt{n'\tilde{n}'}} a'^{\alpha'}_{-n'}\tilde{a}'^{\beta'}_{-\tilde{n}'}|\mathcal{P}'\rangle \tag{22}$$

where a and $\tilde{a}$ are given precisely by eqs.(8), (9), while a' and $\tilde{a}'$ are defined by the same formulas with the substitution $e^\mu \to \mathrm{e}'^\mu$, $\xi^\alpha_i \to \xi'^\alpha_i$.

Making use of eq.(18) one writes the momentum conservation relation,

$$Q^\mu = (\mathcal{P}^\mu - \mathcal{P}'^\mu) - \frac{2(n+\tilde{n})}{(e\mathcal{P})}e^\mu + \frac{2(n'+\tilde{n}')}{(e'\mathcal{P}')}e'^\mu \tag{23}$$

Note that it follows from this relation and eq.(16) that Q^1 is quantized in units $1/L$,

$$Q^1 = \frac{r}{L}, \qquad r = 0, \pm 1, \dots \tag{24}$$

as it should be for compact X^1 ($\mathcal{P}$ and $\mathcal{P}'$ are also quantized [18]).

In our case of light-like Q^μ, eq.(23) implies that $\mathcal{P} = \mathcal{P}'$ only when Q^μ, e^μ and e'^μ are aligned. For other, non-exceptional Q^μ the smooth part of the final string state carries non-zero recoil momentum. In $(1+1)$-dimensional language this means that the emission of a baby universe with non-exceptional global quantum numbers Q^μ occurs only when spatially homogeneous modes of the field $X^\mu(\sigma,\tau)$ are excited.

This process involves the interaction with the entire parent universe, and we will see shortly that the corresponding amplitude is suppressed for universes of large size L.

We consider the technically simplest case

$$e^{\mu} = e'^{\mu}$$

$$\boldsymbol{\xi}^{\alpha} = \boldsymbol{\xi}'^{\alpha}$$

This case includes both the situation with recoil, $\mathcal{P} \neq \mathcal{P}'$, and the exceptional situation without recoil, when

$$Q^{\mu} = -\left[\frac{2(n+\tilde{n})}{(e\mathcal{P})} - \frac{2(n'+\tilde{n}')}{(e\mathcal{P}')}\right] e^{\mu}$$

Furthermore, we take

$$e^{1} = 0 \tag{25}$$

$$\mathcal{P}^{1} = \mathcal{P}'^{1} = 0 \tag{26}$$

so that

$$Q^{1} = 0 \tag{27}$$

and also assume that

$$e^{\mu}\zeta_{\mu\nu} = e^{\nu}\zeta_{\mu\nu} = 0 \tag{28}$$

These restrictions are purely technical; they simplify the calculations considerably. Note that eq.(15) implies then

$$\tilde{n} = n, \qquad \tilde{n}' = n'$$

Finally, we consider the case

$$\alpha' \neq \alpha$$

$$\beta' \neq \beta \tag{29}$$

which, in $(1+1)$-dimensional language, means that the "particles" change their $SO(D-2)$ global quantum numbers when interacting with the baby universe.

The calculation of the amplitude (20) is then straightforward. The integration over zero modes leads to momentum conservation, eq.(23), up to normalization factors about which we will have to say more later. The non-zero modes give rise to the product of left and right factors,

$$A = \kappa\zeta_{\mu\nu}\xi_{i}^{\alpha}\xi_{i'}^{\alpha'}\xi_{j}^{\beta}\xi_{j'}^{\beta'}A_{L,ii'}^{\mu}A_{R,jj'}^{\nu} \tag{30}$$

The calculation of A_L is outlined in Appendix. One finds

$$A^\mu_{L,ii'} = \frac{1}{2nn'} \frac{\Gamma\left(n+1-\frac{n}{(e\mathcal{P})}(eQ)\right)}{\Gamma(n)\Gamma\left(2-\frac{n}{(e\mathcal{P})}(eQ)\right)} \frac{\Gamma\left(n'+1+\frac{n'}{(e\mathcal{P}')}(eQ)\right)}{\Gamma(n')\Gamma\left(2+\frac{n'}{(e\mathcal{P}')}(eQ)\right)} \times$$

$$\left(\frac{\mathcal{P}^\mu_L + \mathcal{P}'^\mu_L}{8\sqrt{nn'}} U^i V^{i'} - \delta^{\mu i}\sqrt{\frac{n}{n'}} V^{i'} - \delta^{\mu i'}\sqrt{\frac{n'}{n}} U^i\right) \tag{31}$$

where

$$U^i = \frac{n}{n - \frac{n}{(e\mathcal{P})}(eQ)} \left[1 - \frac{n}{(e\mathcal{P})}(eQ)\right] \left[-Q^i + \mathcal{P}^i_L \frac{(eQ)}{(e\mathcal{P})}\right] \tag{32}$$

$$V^{i'} = \frac{n'}{n' + \frac{n'}{(e\mathcal{P}')}(eQ)} \left[1 + \frac{n'}{(e\mathcal{P}')}(eQ)\right] \left[Q^{i'} - \mathcal{P}'^{i'}_L \frac{(eQ)}{(e\mathcal{P}')}\right] \tag{33}$$

The factor $A^\nu_{R,jj'}$ is obtained from these expressions by substituting $\mathcal{P}_L, \mathcal{P}'_L \to \mathcal{P}_R, \mathcal{P}'_R$, $i, i' \to j, j'$, $\mu \to \nu$.

We are interested in the limit of large L and finite n/L, n'/L and Q^μ. The initial string is taken to be at rest, $\mathcal{P} = (M_0, 0, \ldots, 0)$. In this limit one has $(e\mathcal{P}) = (e\mathcal{P}') = -M_0 = -2L$, $\mathcal{P}^i_L = \mathcal{P}'^i_L = 2L\delta^{i,1}$, and using Stirling's formula one obtains

$$A^\mu_{L,ii'} = \frac{1}{2\Gamma\left(2+\frac{n}{2L}(eQ)\right)\Gamma\left(2-\frac{n'}{2L}(eQ)\right)} \left(\frac{\mathcal{P}^\mu_L + \mathcal{P}'^\mu_L}{8\sqrt{nn'}} U^i V^{i'} - \delta^{\mu i}\sqrt{\frac{n}{n'}} V^{i'} - \delta^{\mu i'}\sqrt{\frac{n'}{n}} U^i\right) \times$$

$$\exp\left[(eQ)\left(\frac{n}{2L}\ln n - \frac{n'}{2L}\ln n'\right)\right]$$

with

$$U^i = \left[1 + \frac{n}{2L}(eQ)\right]\left[-Q^i - \delta^{i1}(eQ)\right]$$

$$V^{i'} = \left[1 - \frac{n'}{2L}(eQ)\right]\left[Q^i + \delta^{i1}(eQ)\right]$$

We see that the amplitude (30) non-trivially depends on the parameters of the "particles" (their $SO(D-2)$ flavor and $(1+1)d$ bare energy n/L), and that it behaves at $n \sim n' \sim L$ as

$$\left(\frac{1}{L}\right)^{-\frac{n-n'}{L}(eQ)}$$

Note that the target space energy conservation implies in the limit of large L that

$$Q^0 = M_i - M_f = \frac{2(n-n')}{L}$$

(see eq.(19)), so that the suppression factor is

$$\left(\frac{1}{L}\right)^{-\frac{1}{2}Q^0(eQ)} \tag{34}$$

The amplitude is finite in the limit $L \to \infty$ only when there is no recoil into zero modes, i.e., when $\mathcal{P} = \mathcal{P}'$ and $(eQ) = 0$; otherwise $(eQ) = -Q^0 + \boldsymbol{eQ} < 0$, and the amplitude vanishes. This confirms the expectations outlined in Introduction and in the beginning of this section.

Let us finally count the remaining powers of L in the probability of the graviton emission. Let all states have the normalization appropriate for compact X^1, say, for graviton $\langle Q'|Q\rangle \propto \delta_{r,r'}$ with no L-dependent factors (r is defined in eq.(24)).Then the normalization factors for both string states and graviton give rise to the factor $L^{-3/2}$ in the amplitude, while integration over the zero mode X^1 in eq.(20) produces the factor L. This leaves the factor L^{-1} in the probability. The energy-dependent factors in the emission probability, $1/E_i E_f$, give another factor $M_0^{-2} \sim L^{-2}$. The density of states of the graviton and final string produce the factor $L^2 dQ^1 d(n'/L)$, so that the emission probability is proportional to $L^{-1} d(n'/L)$. This is precisely the volume dependence of the probability of scattering of two "particles" in $(1+1)$ dimensions with finite "momenta" n/L, given that the states of these particles are normalized to contain one particle in volume L, see eq.(14). We conclude that apart from the factor (34) there is no further suppression of the probability of scattering of two "particles" in the large $(1+1)d$ universe with induced creation of a baby universe.

Since the transition amplitude is unsuppressed at large L only for exceptional momenta, i.e., only in the zero measure region of phase space, the emission probability vanishes too fast in the limit $L \to \infty$. Thus, the process considered in this section does not lead to the loss of quantum coherence in the $(1+1)d$ universe of infinite size. As discussed above, the origin of this suppression is essentially kinematical, and we do not expect such a suppression in situations when the excitation of zero modes is not required by kinematics. We support this expectation further in section 4 by considering the emission of a tachyon without recoil into zero modes. Before doing so, let us briefly discuss the amplitude of the tachyon emission in the case when the zero modes *are* excited. It will be instructive to see that tachyons behave qualitatively in the same way as gravitons in this case.

3.2 Emission of tachyon with recoil into zero modes

Let us consider the amplitude of the emission of a tachyon with target space momentum Q,

$$\kappa\langle f| : \mathrm{e}^{-iQX(0)} : |i\rangle \tag{35}$$

where the states $|i\rangle$ and $|f\rangle$ are defined by eqs.(21) and (22). In this section we again study the particular case

$$e^{\mu} = e'^{\mu}$$

$$\boldsymbol{\xi}^{\alpha} = \boldsymbol{\xi}'^{\alpha}$$

The target space momentum conservation, eq.(23), implies that in this case the zero modes are necessarily excited,

$$\mathcal{P}^{\mu} \neq \mathcal{P}'^{\mu}$$

We again impose our restrictions (25), (26) and (29) to simplify the calculations.

The evaluation of the tachyon amplitude (35) is similar (and simpler) than that outlined in Appendix. One finds, again up to normalization factors due to zero modes,

$$A = \kappa \xi_i^{\alpha} \xi_{i'}^{\alpha'} \xi_j^{\beta} \xi_{j'}^{\beta'} A_{L,ii'} A_{R,jj'} \tag{36}$$

where

$$A_{L,ii'} = -\frac{1}{4\sqrt{nn'}} \frac{\Gamma\left(n - \frac{n}{(e\mathcal{P})}(eQ)\right)}{\Gamma(n)\Gamma\left(1 - \frac{n}{(e\mathcal{P})}(eQ)\right)} \frac{\Gamma\left(n' + \frac{n'}{(e\mathcal{P}')}(eQ)\right)}{\Gamma(n')\Gamma\left(1 + \frac{n'}{(e\mathcal{P}')}(eQ)\right)} \times$$
$$\left(Q^i - \frac{(eQ)}{(e\mathcal{P})}\mathcal{P}_L^i\right)\left(Q^{i'} - \frac{(eQ)}{(e\mathcal{P}')}\mathcal{P}_L'^{i'}\right)$$

In the intersting limit of large L and finite n/L, n'/L and Q (and the initial string at rest) this expression has the following asymptotics,

$$A_{L,ii'} = -\frac{1}{4\Gamma\left(1+\frac{n}{2L}(eQ)\right)\Gamma\left(1-\frac{n'}{2L}(eQ)\right)}\left[Q^i+\delta^{i1}(eQ)\right]\left[Q^{i'}+\delta^{i'1}(eQ)\right]\times$$

$$\exp\left[\left(-\frac{1}{2}+\frac{n}{2L}(eQ)\right)\ln n+\left(-\frac{1}{2}-\frac{n'}{2L}(eQ)\right)\ln n'\right]$$

Therefore, the amplitude is of order

$$A \sim \left(\frac{1}{L}\right)^{2-\frac{n-n'}{L}(eQ)} \sim \left(\frac{1}{L}\right)^{2-\frac{1}{2}Q^0(eQ)} \tag{37}$$

Given that $Q^2 = 8$ for tachyon, the exponent in this expression is always positive, and tends to zero at large $|\boldsymbol{Q}|$ and $\boldsymbol{Q} \propto \boldsymbol{e}$. Precisely in this regime the recoil momentum $(\mathcal{P}' - \mathcal{P})$ tends to zero. We again find that the amplitude is unsuppressed only for exceptional tachyon momenta, when the zero modes of the large string are not excited.

To conclude this section, we point out that the suppression factors like (34) or (37) are not entirely new in string theory. Similar suppression appears in the form-factor of the leading trajectory of large mass $M \sim L$ [13]. This suppression should be generic for macroscopic strings and should allow for the interpretation as coming from the interaction of graviton or tachyon with the entire string.

4 Emission of tachyon without recoil into zero modes

When the conservation of the target space momentum in the process of emission of a baby universe does not require the excitation of zero modes of the fields $X^\mu(\sigma,\tau)$ in the parent one, one expects no suppression of the corresponding amplitude at large L. In flat target space-time this is possible when the emitted microscopic string state is a tachyon. So, let us consider the amplitude

$$\langle\mathcal{P}|a'^{\alpha'}_{n'}\tilde{a}'^{\beta'}_{\tilde{n}'} : \mathrm{e}^{-iQX(0)} : a^{\alpha}_{-n}\tilde{a}^{\beta}_{-\tilde{n}}|\mathcal{P}\rangle \tag{38}$$

where $\mathcal{P} = (M_0, 0, \ldots, 0)$ for both initial and final macroscopic strings, and the DDF operators relevant to the initial and final states are constructed with different sets of vectors $(e^\mu, \boldsymbol{\xi}^\alpha)$ and $(e'^\mu, \boldsymbol{\xi}'^\alpha)$, each obeying the relations (5), (6), (7), i.e.,

$$e^0 = e'^0 = 1 \tag{39}$$

$$e^2_\mu = e'^2_\mu = 0 \tag{40}$$

$$\boldsymbol{\xi}^\alpha \boldsymbol{\xi}^\beta = \boldsymbol{\xi}'^\alpha \boldsymbol{\xi}'^\beta = \delta^{\alpha\beta}$$
$$\boldsymbol{\xi}^\alpha \boldsymbol{e} = \boldsymbol{\xi}'^\alpha \boldsymbol{e}' = 0 \tag{41}$$

The target space momentum conservation, eq.(23), in this case reads

$$Q^\mu = \frac{2(n+\tilde{n})}{M_0} e^\mu - \frac{2(n'+\tilde{n}')}{M_0} e'^\mu \tag{42}$$

We evaluate the amplitude (38) in the general case, without imposing any restrictions like eqs.(25), (26) or (29). The oscillator operator algebra involved in the calculation is similar to that outlined in Appendix. One finds, again up to normalization factors coming from zero modes,

$$A = \kappa \xi_i^\alpha \xi_{i'}^{\alpha'} \xi_j^\beta \xi_{j'}^{\beta'} A_{L,ii'} A_{R,jj'} \tag{43}$$

where

$$A_{L,ii'} = \frac{1}{\sqrt{nn'}} \int \frac{dz}{2\pi} \frac{dz'}{2\pi} \frac{1}{z^{n+1} (z')^{n'+1}} (1-z)^{\frac{n}{(e\mathcal{P}_L)}(eQ)} \times$$
$$(1-z')^{-\frac{n'}{(e'\mathcal{P}_L)}(e'Q)} (1-zz')^{-\frac{4nn'}{(e\mathcal{P}_L)(e'\mathcal{P}_L)}(ee')} \left[U^i(z) V^{i'}(z') + \delta^{ii'} \frac{zz'}{(1-zz')^2} \right] \tag{44}$$

where the integration contours are small circles around the origin in the complex plane. Here

$$U^i(z) = \frac{\mathcal{P}_L^i}{2} + \frac{Q^i}{2} \frac{z}{1-z} - \frac{2n'}{(e'\mathcal{P}_L)} e'^i \frac{zz'}{1-zz'}$$
$$V^{i'}(z') = \frac{\mathcal{P}_L^{i'}}{2} - \frac{Q^{i'}}{2} \frac{z'}{1-z'} - \frac{2n}{(e\mathcal{P}_L)} e^{i'} \frac{zz'}{1-zz'}$$

We now recall the relations (17), (38), (40) and (41) and also use $(e\mathcal{P}) = -M_0$. The target space momentum conservation with $Q^2 = 8$ gives

$$\frac{4nn'}{(e\mathcal{P}_L)(e'\mathcal{P}_L)}(ee') = -1$$
$$\frac{n}{(e\mathcal{P}_L)}(eQ) = -1$$
$$\frac{n'}{(e'\mathcal{P}_L)}(e'Q) = 1$$

These relations make the integration in eq.(44) particularly simple. We obtain for $n' < n$

$$A_{L,ii'} = \left(e^{i'} e'^i \frac{(n+\tilde{n})}{M_0} \frac{(n'+\tilde{n'})}{M_0} + \delta^{ii'} \right) \sqrt{\frac{n'}{n}} \tag{45}$$

Equivalently, $A_{L,ii'}$ can be written in the following form (we again use eqs.(40), (41) and (42))

$$A_{L,ii'} = \left(\frac{1}{4}Q^i Q^{i'} + \delta^{ii'}\right)\sqrt{\frac{n'}{n}}$$

The right factor is obtained in a similar way,

$$A_{R,jj'} = \left(\frac{1}{4}Q^j Q^{j'} + \delta^{jj'}\right)\sqrt{\frac{n'}{n}}$$

We see that the amplitude (43) is finite as $L \to \infty$.

The remaining L-dependent factors in the emission probability are counted in the same way as in the end of sect. 3.1, with the same conclusion. It can be seen, however, that the final states of the type considered in this section do not span entire phase space, i.e., the emission amplitude is unsuppressed only in a zero measure region of the phase space. This means that at the level of two-"particle" DDF excitations of the final macroscopic string, the emission probability is still suppressed at large L. We think, however, that our results indicate that the *total* emission probability is finite at large L, i.e., the loss of quantum coherence is likely to occur at finite rate in $(1+1)$-dimensional universe of large size.

5 Conclusion

We have found in this paper that string theory viewed as the theory of $(1+1)d$ universes meets the expectations on the emission of baby universes due to interactions of particles in the parent universe. We have argued, on the basis of understanding of strings from D-dimensional point of view, that processes involving baby universes/wormholes not only induce c-number α-parameters in the $(1+1)d$ action, but also lead to the loss of quantum coherence for $(1+1)d$ observer in the parent universe.

We have considered strings in *flat* D-dimensional space-time, and restricted ourselves to particular final states of the macroscopic string, with only *two* DDF "particles". The consequence of these limitations was that only tachyons, or gravitons with very exceptional momenta, can be emitted without recoil into zero modes. In both cases the recoil was absent only in a zero measure region of phase space, so we were unable to show that the emission rate, i.e., the rate of the loss of quantum coherence, is finite for large $(1+1)d$ universes. Also, the necessity to consider tachyons appears unsatisfactory. We think that the latter peculiarity is not too relevant for

our purposes: indeed, we have seen in sect.3.2 that tachyons and gravitons behave similarly in our context. One possibility to improve our analysis would be to study the *total* probability of the graviton emission by macroscopis (super)strings *in non-trivial D-dimensional background fields*, when the kinematical constraints are not so restrictive.

It is not obvious that the simple physical picture evident in string model of $(1+1)d$ universes can be extrapolated to (3+1) dimensional case. In string theory, there exists a natural causal structure of the D-dimensional target space. It is not clear whether such a structure is inherent in the superspace of $(3+1)d$ theory. However, it is feasible that the notion of baby universes propagating in (mini-)superspace, which was crucial for our discussion of the loss of quantum coherence, exists also in $(3+1)d$ theory: some of the known examples of wormhole solutions in four dimensions [3, 20, 21, 22], being appropriately continued from euclidean time, describe baby universes that branch off and then evolve non-trivially in their intrinsic time (either shrink to singularity or expand to large sizes). These may be candidates for baby universes travelling in superspace.

Finally, let us point out that understanding, in the context of $(1+1)d$ theory, of processes involving baby universes may be of interest for the solution of the information problem in black hole physics, in view of suggestions that baby universes/wormholes may become important at the late stages of the black hole evaporation (for discussion and references see, e.g., refs.[23, 24, 25]).

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Appendix. Amplitude of graviton emission

Apart from the zero mode integral, the amplitude (20) decomposes into left and right parts, as written in eq.(30). Making use of the explicit form of the DDF operators and eqs.(27) and (28), the left factor can be written in the following form,

$$A^{\mu}_{L,ii'} = \frac{1}{\sqrt{nn'}} \int_0^{\pi} \frac{d\sigma_+}{\pi}\frac{d\sigma'_+}{\pi}\ \langle 0|\mathrm{e}^{2in'\sigma'_+} \exp\left[-\frac{2n'}{(e\mathcal{P}')}\sum_{q>0}\frac{1}{q}e_{\lambda}\alpha^{\lambda}_{q}\mathrm{e}^{-2iq\sigma'_+}\right] \times$$

$$\left(\frac{1}{2}\mathcal{P}'^{i'}_{L} + \sum_{k>0}\alpha^{i'}_{k}\mathrm{e}^{-2ik\sigma'_+}\right) \exp\left[\frac{1}{2}\sum_{k<0}\frac{1}{k}Q_{\lambda}\alpha^{\lambda}_{k}\right] \left(\frac{\mathcal{P}^{\mu}_{L}+\mathcal{P}'^{\mu}_{L}}{4} + \sum_{k\neq 0}\alpha^{\mu}_{k}\right) \times$$

$$\exp\left[\frac{1}{2}\sum_{k>0}\frac{1}{k}Q_{\lambda}\alpha^{\lambda}_{k}\right] \mathrm{e}^{-2in\sigma_+} \exp\left[\frac{2n}{(e\mathcal{P})}\sum_{q<0}\frac{1}{q}e_{\lambda}\alpha^{\lambda}_{q}\mathrm{e}^{-2iq\sigma_+}\right] \left(\frac{1}{2}\mathcal{P}^{i}_{L} + \sum_{k<0}\alpha^{i}_{k}\mathrm{e}^{-2ik\sigma_+}\right)|0\rangle$$

Moving then the first exponential factor to the right with the use of eqs.(28) and (7), one picks up the factor

$$\exp\left[\frac{n'}{(e\mathcal{P}')}(eQ)\sum_{q>0}\frac{1}{q}\mathrm{e}^{-2iq\sigma'_+}\right] = \left(1-\mathrm{e}^{-2i\sigma'_+}\right)^{-\frac{n'}{(e\mathcal{P}')}(eQ)}$$

Similarly, moving the last exponential factor to the left produces a factor

$$\left(1-\mathrm{e}^{2i\sigma_+}\right)^{\frac{n}{(e\mathcal{P})}(eQ)}$$

Moving the first Q-dependent exponential factor to the left leads to adding a term

$$-\frac{1}{2}Q^{i'}\sum_{k>0}\mathrm{e}^{-2ik\sigma'_+} = -\frac{1}{2}Q^{i'}\frac{\mathrm{e}^{-2i\sigma'_+}}{1-\mathrm{e}^{-2i\sigma'_+}}$$

to $\frac{1}{2}\mathcal{P}'^{i'}_{L}$ in the corresponding parenthesis. Similarly, a term

$$\frac{1}{2}Q^{i}\frac{\mathrm{e}^{2i\sigma_+}}{1-\mathrm{e}^{2i\sigma_+}}$$

is added to $\frac{1}{2}\mathcal{P}^{i}_{L}$ after moving the second Q-dependent exponential factor to the right. The remaining matrix element

$$\langle 0|\left(\frac{1}{2}\mathcal{P}'^{i'}_{L} - \frac{1}{2}Q^{i'}\frac{\mathrm{e}^{-2i\sigma'_+}}{1-\mathrm{e}^{-2i\sigma'_+}} + \sum_{k>0}\alpha^{i'}_{k}\mathrm{e}^{-2ik\sigma'_+}\right)\left(\frac{\mathcal{P}^{\mu}_{L}+\mathcal{P}'^{\mu}_{L}}{4} + \sum_{k\neq 0}\alpha^{\mu}_{k}\right) \times$$

$$\left(\frac{1}{2}\mathcal{P}^{i}_{L} + \frac{1}{2}Q^{i}\frac{\mathrm{e}^{2i\sigma_+}}{1-\mathrm{e}^{2i\sigma_+}} + \sum_{k<0}\alpha^{i}_{k}\mathrm{e}^{-2ik\sigma_+}\right)|0\rangle$$

is straightforward to evaluate. Collecting all factors, and denoting

$$\mathrm{e}^{2i\sigma_+} = z \qquad \mathrm{e}^{-2i\sigma'_+} = z'$$

one obtains at $i \neq i'$ (see eq.(29))

$$A^{\mu}_{L,ii'} = \frac{1}{\sqrt{nn'}} \int \frac{dz}{2\pi} \frac{dz'}{2\pi} \frac{1}{z^{n+1}} \frac{1}{(z')^{n'+1}} \times$$

$$\left[\frac{\mathcal{P}^{\mu}_{L} + \mathcal{P}'^{\mu}_{L}}{16} U^{i}(z) V^{i'}(z') + \frac{1}{2} \delta^{i'\mu} \frac{z'}{(1-z')^{2}} U^{i}(z) + \frac{1}{2} \delta^{i\mu} \frac{z}{(1-z)^{2}} V^{i'}(z') \right]$$

where the integration contours are small circles around the origin in complex plane and

$$U^{i}(z) = \mathcal{P}^{i}_{L} + Q^{i} \frac{z}{1-z}$$

$$V^{i'}(z') = \mathcal{P}'^{i'}_{L} - Q^{i'} \frac{z'}{1-z'}$$

The integrals over z and z' factorize and have the form

$$\int \frac{dz}{2\pi} \frac{1}{z^{N+1}} (1-z)^{\alpha} = (-1)^{N} \frac{\Gamma(\alpha+1)}{\Gamma(N+1)\Gamma(\alpha-N+1)}$$

Equation (31) is then obtained by simple algebra with the use of the properties of gamma-function.

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